FYUGP/1st Sem/23(NEP)

2023

Four Year Under Graduate Programme (FYUGP)

1st Semester Examination (Under NEP)

MATHEMATICS (Major)

Paper Code : MTMMJ MC-01

(Calculus & Geometry)

Full Marks : 40 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

(Marks - 10)

1. Answer any *five* questions : $2\times5=10$

(a) Use Leibnitz's formula to find the nth order derivative of x^3e^{ax}.

(b) Show that $\lim_{x\to0}\frac{1}{x}\sin\left(\frac{1}{x}\right)$ doest not exist.

(c) A function $f:\mathbb{R}\to\mathbb{R}$ is defined by

$$f(x)=\cos\frac{1}{x},\quad x\neq0,$$
$$=0,\quad x=0$$

Prove that f is not continuous at 0.

P.T.O.

(d) Find the length of the curve $y = \frac{a}{2}\left(e^{x/a} + e^{-x/a}\right)$ between the points $x = 0$ and $x = x$.

(e) Find the centre and the radius of the sphere

$$x^2 + y^2 + z^2 - 2x + 4y - 6z = 11.$$

(f) Find the equation of the normal at the point (1, 1) to the hyperbola $x^2 - 2y^2 + x + y = 1$.

(g) Find the midpoint of the chord of $y^2 = 4x$ which lie on the straight line $y = 5x - 1$.

Group - B

(Marks - 30)

Answer any *six* questions : 5×6=30

2. If $y = e^{ax} \cos^2 bx$, find y_n.

3. Prove that

$$B(x, y) = \int_0^1 \frac{t^{x-1} + t^{y-1}}{(1+t)^{x+y}} dt = \frac{1}{2}\int_0^\infty \frac{t^{x-1} + t^{y-1}}{(1+t)^{x+y}} dt$$

4. If $I_n = \int_0^1 x^n \tan^{-1} x \, dx$, then show that

$$(n+1) I_n = \frac{\pi}{2} - \frac{1}{n} - (n-2) I_{n-2}.$$

5. Find the equation of the cubic which has the same asymptotes as the curve $x^3-6x^2y+11xy^2-6y^3+x+y+1=0$ and which passes through $(0, 0)$, $(1, 0)$ and $(0, 1)$.

6. Show that the surface area of the solid generated by revolving the cycloid $x=a(\theta-\sin\theta)$, $y=a(1-\cos\theta)$ about the line $y=0$, is $\frac{64}{3}\pi a^2$.

7. Show that the plane $y+3=0$ intersects the hyperbolic paraboloid $\frac{x^2}{5}-\frac{y^2}{4}=6z$ in a parabola. Find the vertex and the length of the latus rectum of the parabola.

8. Prove that the two conics $\frac{l_1}{r}=1-e_1\cos\theta$ and $\frac{l_2}{r}=1-e_2\cos(\theta-\alpha)$ will touch one another, if $l_1^2(1-e_2^2)+l_2^2(1-e_1^2)=2l_1l_2(1-e_1e_2\cos\alpha)$.

9. Show that the circles $x^2+y^2+z^2-2x+3y+4z-5=0$, $5y+6z+1=0$ and $x^2+y^2+z^2-3x-4y+5z-6=0$, $x+2y-7z=0$ lie on the same sphere, whose equation is $x^2+y^2+z^2-2x-2y-2z-6=0$.

P.T.O.

10. Reduce the equation $3x^2-24y^2+8z^2+16yz-10zx-14xy+22y+2z-4=0$ to the canonical form and find its axis.

2023

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MATHEMATICS (Major)

Paper Code : MTMMJ MC-02

(Algebra)

Full Marks : 40 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

(Marks - 10)

1. Answer any *five* questions : 2×5=10

(a) Find the principal value of $(1+i)^{1-i}$.

(b) Apply Descartes' rule of signs to examine the nature of the roots of the equation $x^6+x^4+x^2+x+3=0$.

(c) Find the minimum value of $x^3+y^3+z^3$ when x, y, z are real and positive and are related by $9x+16y+25z=15$.

P.T.O.

A

(d) The eigen values of an $(n+1)\times(n+1)$ matrix A are 0 and the n-th roots of unity. Prove that

$$2I=\left\{I+\frac{1}{2^n-1}\left(2^{n-1}A+2^{n-2}A^2+\cdots+2A^{n-1}+A^n\right)\right\}(2I-A).$$

(e) Show that the quadratic form

$$x_1^2+x_2^2+x_3^2-x_1x_2-x_2x_3-x_3x_1$$

is positive semi-definite.

(f) Let $f:\mathbb{R}\to\mathbb{R}$ be defined by $f(x)=\cos(5x+2)$ for all $x\in\mathbb{R}$. Prove that f is not bijective.

(g) Solve the recurrence relation

$$f_n=f_{n-1}+f_{n-2}$$

with $f_0=0$ and $f_1=1$.

Group - B

(Marks - 30)

Answer any *six* questions : 5×6=30

2. If $A=\begin{pmatrix}1&0&0\\1&0&1\\0&1&0\end{pmatrix}$, show that for every integer $n\geq 3$, $A^n=A^{n-2}+A^2+I$. Hence evaluate A^{50}.

3. For what values of λ and μ the following system of equations

$$-4x+2y-9z=2,$$
$$3x+4y+\ z=5,$$
$$3x+4y+\lambda z=\mu$$

has no solution, a unique solution and an infinite number of solutions? 5

4. (a) Solve : $\exp(z)=1+\sqrt{3}i$, where $z=x+iy$. 3

(b) Find the general values of i^i. 2

5. Prove that the roots of the equation $x^3-6x-4=0$ are $-2, 2\sqrt{2}\cos\frac{\pi}{12}, 2\sqrt{2}\cos\frac{7\pi}{12}$. 5

6. (a) Show that the equation

$$(x-a)^3+(x-b)^3+(x-c)^3+(x-d)^3=0,$$

where a, b, c, d are not all equal, has only one real root. 3

(b) If α, β, γ be the roots of the cubic equation $ax^3+3bx^2+3cx+d=0$, then find the value of the expression $(a\alpha+b)(a\beta+b)(a\gamma+b)$ in terms of the co-efficients a, b, c, d. 2

P.T.O.

7. Show that the equation

$$x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1 = 0$$

is a reciprocal equation. Write down its type and find its solution. 5

8. If $f: A \to B$ and $g: B \to C$ be two mappings and $(g \circ f)$ is one-one and onto, then show that f is one-one and g is onto.

9. (a) Using elementary row operations, reduce the matrix A to its Echelon form and hence find its rank, where

$$A = \begin{pmatrix} 2 & 1 & 3 & -2 \\ 2 & -1 & 5 & 2 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$ 3

(b) Find the generating function, the general term and the sum of the first n terms of the recurring series $2 + 7x + 25x^2 + 91x^3 + \cdots$. 2

0. (a) Show that $8xyz < (1-x)(1-y)(1-z) < \frac{8}{27}$ when $x + y + z = 1$. 3

(b) If z be a complex number and $\frac{z+1}{z-i}$ be purely imaginary, then show that z lies on the circle whose centre is at the point $\frac{1}{2}(-1+i)$ and radius is $\frac{1}{\sqrt{2}}$.

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1st Semester Examination (Under NEP)

MATHEMATICS (Major)

Paper Code : MTMMJ SEC-01

(Number Theory & Boolean Algebra)

Full Marks : 40 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

(Marks - 10)

1. Answer any *five* questions : $2\times5=10$

(a) Find the remainder when 7^{350} is divisible by 11.

(b) Prove that $\gcd(a, a+2)=1$ or 2, for every integer a.

(c) Show that in a Boolean algebra B,

$$(a'+ab)'=ab' \text{ for all } a, b \in B.$$

(d) Write the input-output table for a NAND gate.

P.T.O.

(e) In a Boolean algebra $(B, +, \cdot, ')$, prove that $O' = 1$.

(f) Which elements of the poset ({2, 4, 5, 10, 12, 20, 25}, 1) are maximal and which are minimal?

(g) Find the decimal representation of the number $(11010111)_2$.

Group - B

(Marks - 30)

Answer any *six* questions : 5×6=30

2. Show by induction that $2n+1 < n^3$ for all integers $n \geq 2$. 5

3. Solve the system of linear congruence :

$$x \equiv 2 \pmod 3$$

$$x \equiv 3 \pmod 5$$

$$x \equiv 4 \pmod 7.$$ 5

4. Find integers u and v satisfying

$$30u + 72v = 12.$$ 5

5. In the 12-digit UPC (Universal Product Code) 0724 8903 004-*, of a product, what is the 12th digit, i.e., the check digit? 5

6. Prove that for any two given integers a and b with $b>0$, there exist unique integers q and r such that

$$a = bq + r \text{ where } 0 \le r < b.$$ 5

7. Use a Karnaugh-map to find the minimized sum-of-products Boolean expression of the Boolean expression $xyz + xyz' + xy'z' + x'yz + x'yz'$. 5

8. Draw the circuit represented by the Boolean function

$$a(a'+b)+b(b+c)+b.$$

Simplify the function and draw the simplified circuit.

3+2

9. (a) Define distributive lattice.

(b) In a distributive lattice $(L, \le)$, prove that $a \wedge b = a \wedge c$ and $a \vee b = a \vee c$ imply that $b = c$, for all $a, b, c \in L$. 1+4

10. (a) If P be a prime and K be a positive integer then prove that

$$\phi(P^K) = P^K\left(1-\frac{1}{P}\right)$$

(b) Determine the integer in the unit place of $17^{17^{17}}$.

3+2

P.T.O.